Toward Standard Figures-of-Merit for Spatial and Quasi-Optical Power-Combined Arrays

Mark Gouker

***Abstract*—A consistent set of figures-of-merit is proposed for the standard characterization of spatial and quasioptical power-combined arrays. A new figure-of-merit, the effective transmitter power, is presented along with slightly modified definitions of standard figures-of-merit. The definitions of these figures-of-merit have been chosen to more directly compare the performance of spatial and quasioptical power-combined arrays with one another and with conventional circuit power-combined transmitters and amplifiers.**

I. Introduction

The development of quasioptical[1] power combining [2], [3] has generated significant interest as an alternative approach for moderate power solid-state oscillators and amplifiers. In addition to generating an explosion of activity in quasioptical power-combining [4], [5], this development has rekindled interest in spatial power-combining techniques that use more traditional antenna array configurations [6]–[9]. The rapid growth in this field has led to a number of different definitions for the figures-of-merit which quantify the performance of these arrays. In this paper standardization of some of these figures-of-merit is proposed.

The overriding guideline in the proposed definitions of the figures-of-merit is to account for the loss mechanisms which are present in the spatial power-combining technique.[2] The inclusion of these losses enables a more direct comparison between different spatial power-combined arrays and a comparison to conventional circuit combining techniques.

II. Figures-of-Merit

A. Equivalent Isotropic Radiated Power

The most important and unambiguous figure-of-merit for a spatial power-combined array is the equivalent isotropic radiated power, *EIRP*. The *EIRP* is most often determined by measuring the power received by a standard gain horn placed in the far field of the transmitting array. The expression for the *EIRP* is given by

$$EIRP = P_{trans}G_{trans} = \frac{P_{rec}}{G_{rec}}\left(\frac{\lambda_0}{4\pi R}\right)^{-2} \tag{1}$$

where P_{rec} is the power received by the standard gain horn, G_{rec} is the gain of the standard gain horn, and R is the separation distance between the standard gain horn and the array.

Manuscript received October 19, 1994; revised December 28, 1994. This work was supported by the Department of the Army and the Advanced Research Projects Agency Contract F19628-90-C-0002.

The author is with the Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, MA USA.

IEEE Log Number 9412060.

[1]The outputs from a collection of transistors or diodes can be combined at the chip level, in a circuit structure or in free space [1]. The quasi-optical approach is a variation of combining in free space (spatial combining). A distinguishing feature of this approach is that it employs elements originally developed for optical frequencies such as Fabry–Perot cavities and polarizers.

[2]Quasi-optical is implied since it is a subset of the spatial power combining approach. The term quasi-optical will only be used when a reference to this particular type of spatial power combining is made.

B. Effective Transmitter Power

The RF transmitted power, P_{trans} in (1), is also an important figure-of-merit. However, determining the transmitted power with accuracy requires knowledge of the antenna gain for the array, G_{trans} in (1). Unfortunately, the antenna gain for the array is difficult to determine with precision since it requires identification of the loss due to the active devices and the loss due to the radiating elements.

An alternative figure-of-merit, called the **effective transmitter power**, is proposed. The only difference in this new figure-of-merit is that gain for a lossless antenna array, i.e., the directivity of the array, D_{trans}, is used in place of the actual gain. The effective transmitter power, P_{eff}, is defined as

$$P_{eff} = \frac{EIRP}{D_{trans}}. \tag{2}$$

Since the directivity is lossless, using the directivity to calculate the transmitter power instead of using the actual antenna gain accounts for the losses present in the real transmitter. This definition incorporates the dielectric and conductor losses of the radiating elements and the losses due to the deviations from the desired phase and amplitude distribution in the array. There are two major advantages to using the effective transmitter power figure-of-merit in place of the actual transmitter power. First, this definition provides an accurate, unambiguous figure-of-merit that allows direct comparison of the performance of two different spatial power-combined arrays. The proposed definition succinctly quantifies the RF power radiated into the fundamental antenna "mode" (pattern).

Second, this figure-of-merit allows a comparison to conventional circuit combined amplifiers and oscillators. The effective transmitter power is roughly equivalent to the power in the fundamental mode of the output port of a circuit power-combined amplifier or oscillator. In conventional circuit power combining, the loss of the combining circuitry (microstrip or waveguide combiners) is incorporated in the reported output power. Likewise, the losses for the combining circuitry (radiating elements) of a spatial power-combined array are included in the effective transmitter power.

In the majority of arrays, a uniform phase and amplitude is desired since this yields the maximum directivity from the physical aperture. In this case the directivity is most easily calculated using the uniformly illuminated aperture approximation, and it is given by

$$\begin{aligned} D_{trans} &= \frac{4\pi A^{array}}{\lambda_0^2} \\ &= \frac{4\pi A^{uc} n}{\lambda_0^2} \end{aligned} \tag{3}$$

where A^{array} is the physical size of the array, A^{uc} is the physical size of a unit cell (inter-element spacing) in the array, and n is the number of unit cells (elements) in the array. The uniformly illuminated aperture approximation is reasonable provided: 1) both dimensions of the array are greater than two to three wavelengths, or 2) the number of elements per side of the array is much greater than 1, and the spacing between the array elements is small [10]. The second condition is likely to be fulfilled in smaller grid arrays (one to two λ_0 by one to two λ_0).

There are cases, however, where a nonuniform amplitude and/or phase is desired. One example is a quasi-optical amplifier placed at the beam waist of a beam waveguide system. If the array is larger than the beam waist, it will have a nonuniform illumination. Another example is an array designed to feed a reflector antenna system. A nonuniform amplitude and phase may be desired to minimize the

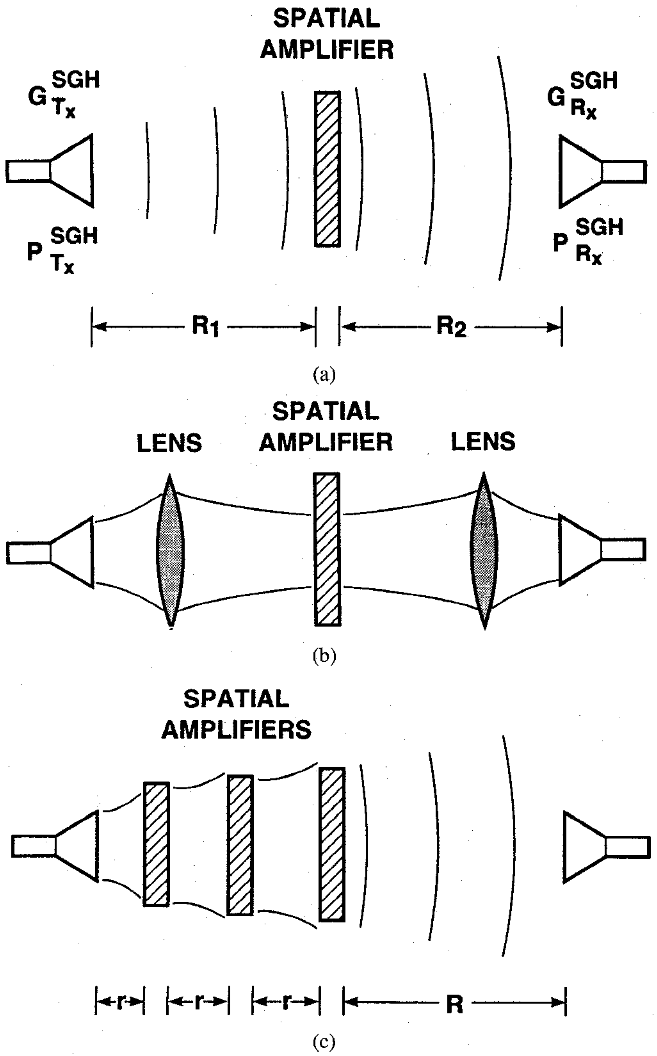


Fig. 1. System configurations for spatial amplifier arrays. (a) Far-field system: $R_1, R_2 > 2D^2/\lambda_0$. (b) Beam waveguide system. (c) Near-field cascade system: $r < D^2/\lambda_0$, $R > 2D^2/\lambda_0$.

spill-over loss in the reflector antenna. In these cases, the directivity will have to be determined through calculation or by measurement. It should be noted that the directivity found by measurement of only the two principle planes is not accurate enough for the calculation of a figure-of-merit. The directivity should be measured by making many conical or great-circle cuts for both the co- and cross-polarization components. The number of cuts depends on the complexity of the antenna pattern [11].

C. System Gain

The third important system figure-of-merit is the total system gain. It is defined as the effective transmitter power divided by the power input into the transmitter. It is given by

$$G_{system} = \frac{P_{eff}}{P_{in}} \tag{4}$$

where P_{eff} is the effective transmitter power of the output stage of the system and P_{in} is the power at the input port of the transmitter. For a spatially-fed array, e.g., [7], [8], [12], [13], the input port might be the waveguide flange of the horn antenna that illuminates the first array in the system. Thus all the losses, such as spill-over and deviation from the desired illumination amplitude and phase, are taken into account. In the case where a spatially-fed array is fed by a distant source, for example, a microwave repeater, the input power should be calculated from the incident power density and the physical aperture of the array. For a circuit-fed array, e.g., [6], [9], the input port might be the coax- to-microstrip transition at the input to the corporate feed network. Thus the loss of the feed-network is taken into account.

D. Power-Added Efficiency (DC-to-RF Conversion Efficiency)

The power-added efficiency (*PAE*) is the fourth quantity that is critical to the characterization of the array performance. For transmitters with high system gain, the *PAE* is essentially the dc-to-RF efficiency, $P_{\text{dc}-\text{RF}}$. It is found from the effective transmitter power of the array minus the power input into the transmitter divided by the dc power supplied to the array, P_{dc}. It is given by

$$\begin{aligned} PAE &= \frac{P_{eff} - P_{in}}{P_{\text{dc}}} \\ &= \eta_{\text{dc}-\text{RF}}\left(1 - \frac{1}{G_{system}}\right). \end{aligned} \tag{5}$$

E. Gain/Scattering Parameters of a Spatial or Quasi-Optical Amplifier Array

The term spatial amplifier will be used to refer to both quasi-optical amplifiers,[3] e.g., [12], [13], and spatially fed/spatially combined arrays,[4] e.g., [7], [8], [14], [15]. In these arrays it is desired to assess the increase in power of the wave front that passes through the array. The system configurations, which utilize the spatial amplifier, can be broken into three categories: far-field systems, beam waveguide systems, and near-field cascade systems, see Fig. 1. In a far-field system, Fig. 1(a), the spatial amplifier is placed in the far-field of its source, and the next element in the system or the designated receiving location is in the far-field of the spatial amplifier. In a beam waveguide system, Fig. 1(b), the spatial amplifier is placed at the beam waist of the beam waveguide. In a near-field cascade system, Fig. 1(c), the spatial amplifier is placed in the near-field of its source, and the next element in the system is placed in the near-field of the spatial amplifier.

For the far-field configuration, a definition of the spatial amplifier gain and a measurement technique have been proposed [13] and have gained acceptance [12], [14], [15]. The spatial amplifier is placed in the far-field of the transmitting standard gain horn, and the power transmitted from the spatial amplifier is measured by placing the receiving standard gain horn in the far-field of the amplifier array as in Fig. 1(a). The expression for the gain can be derived by using the Friis transmission equation and by defining the gain as the ratio of the power radiated from the array to the power incident on the array. Using the approximation for a uniformly illuminated aperture and the physical aperture of the array, A^{array}, the gain of the array is given by [13]

$$\begin{aligned} G^{array} &= \frac{P_{Rx}^{SGH}}{P_{Tx}^{SGH}}\left(\frac{G_{Tx}^{SGH}A^{array}}{4\pi R_1^2}\right)^{-1} \\ &\quad \cdot \left(\frac{G_{Rx}^{SGH}A^{array}}{4\pi R_2^2}\right)^{-1} \end{aligned} \tag{6}$$

where the quantities are defined in Fig. 1(a).

In the beam waveguide configuration, the gain must be measured in a setup which provides a beam waveguide. An added benefit of the beam waveguide setup is that is overcomes many of the practical impediments to performing free-space S-parameter measurements.

[3]Those that contain quasi-optical elements such as beam waveguides and grid polarizers.

[4]Those that are based on conventional antenna designs.

A full set of S-parameters provides a better characterization of the array, because it allows prediction of standing waves between the cascade arrays. However, the presence of the standing waves will modify the antenna input impedance and thus the load presented to the active devices. The effect of the standing waves on the amplifier array performance must be found by modeling at the antenna and active device level. Successful S-parameter measurements have been reported using a beam waveguide arrangement [16], [17].

For spatial amplifiers in a near-field cascade system, there are at least two options in configuring the gain or S-parameter measurement. One can make the assumption that the waves illuminating and radiating from the spatial amplifier are collimated and perform a free-space far-field measurement, as in Fig. 1(a). This assumption ignores reactive near-field coupling between closely spaced arrays, and amplitude and phase variations present in real systems. In other near-field cascade systems, the spatial amplifiers are placed in a custom waveguiding structure with the purpose of more uniformly illuminating the amplifier array. In this case, the gain or S-parameter measurement can be made within this custom waveguide and will require custom calibration standards.

F. Effective Isotropic Power Gain

The effective isotropic power gain (*EIPG*) [7] is the most basic, directly measurable quantity for a spatial amplifier. It is the product of the receive-antenna gain of the spatial amplifier, the gain of the active devices in the array, and the transmit-antenna gain of the spatial amplifier. The *EIPG* is defined as [7]

$$\begin{aligned} EIPG &= G_{Rx}^{array} G^{array} G_{Tx}^{array} \\ &= \frac{P_{Rx}^{SGH}}{P_{Tx}^{SGH}} \frac{1}{G_{Tx}^{SGH} G_{Rx}^{SGH}} \\ &\quad \cdot \left(\frac{\lambda_0}{4\pi R_1}\right)^{-2} \left(\frac{\lambda_0}{4\pi R_2}\right)^{-2} \end{aligned} \tag{7}$$

where G_{Rx}^{array} is the receive-antenna gain of the spatial amplifier, G_{Tx}^{array} is the transmit-antenna gain of the spatial amplifier, and the other quantities are as above, see Fig. 1(a). The *EIPG* has many parallels to the effective isotropic radiated power, *EIRP*, for a transmitting array.

G. Combining Efficiency

The combining efficiency is a quantification of how efficiently the output power from the active devices are combined. It is defined as the ratio of effective transmitter power to the total available power and is given by

$$\eta_{comb} = \frac{P_{eff}}{\sum_{1}^{n} P_{n,avail}} \tag{8}$$

here η_{comb} is the combing efficiency, n is the number of the active devices in the array and $P_{n,avail}$ is the available power from the nth active device. The available power, discussed in more detail below, is defined as the output power from the active device when it is presented with the optimum load impedance (to produce the maximum output power).

There are two loss mechanisms which affect the combing efficiency. The first loss consists of the departures from the desired phase and amplitude of the elements and the resistive and dielectric loss. This loss is taken into account by using the effective transmitter power. The second loss arises from not presenting the optimum load impedance to the active devices. In quasi-optical arrays, the parameters of the active antenna must be properly chosen to present the optimum load impedance to the active device [18]. In more conventional spatial power-combined arrays, the operation of the active device and the antenna are distinct. The impedance of the antenna must be transformed to the optimum load impedance of the active device via an impedance matching network. In both cases, not presenting the optimum load impedance to the active devices is taken into account by using the total available power.

The available power, $P_{n,avail}$, is a function of the dc bias and depends on presenting the optimum load impedance (fundamental and harmonic) to the active device. To determine the available output power of the active device, one must measure it directly through a load-pull measurement. While this technique is accurate, it requires sophisticated instrumentation, and it is not commonly available.

An alternative to the load pull-measurement is to estimate the available output power using idealized models of the active devices. Estimates of the optimum load impedance and available output power for IMPATT didoes is given in [19]. For idealized transistors, the optimum load impedance and available output power for various bias conditions and resistive or tuned load impedances are given in [20]. For MMIC amplifiers, the maximum available output power can be estimated by considering the dc bias of the transistor in the output stage and the equations given in [20].

III. Conclusion

A set of figures-of-merit has been defined to account for the losses present in the spatial power-combining technique. These definitions account for the losses from undesired phase and amplitude variations among the elements, the dielectric and conductor losses of the radiating elements, and the impedance mismatch. The inclusion of these losses in the figures-of-merit permits a more accurate comparison of spatial power-combined arrays with one another and with conventional circuit power-combining techniques.

Acknowledgment

The author would like to thank the following researchers for reading and commenting on this manuscript: K. Chang, R. Compton, T. Itoh, L. Kushner, A. Mortazawi, Z. Popović, G. Rebeiz, D. Rutledge, M. Steer, B. Weikle, and B. York.

References

[1] K. Chang and C. Sun, "Millimeter-wave power combining techniques," *IEEE Trans. Microwave Theory Tech.*, vol. 31, no. 2, pp. 91–107, 1983.

[2] J. W. Mink, "Quasioptical power-combining of solid-state millimeter-wave sources," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 273–279, Feb. 1986.

[3] Z. B. Popović, M. Kim, and D. B. Rutledge, "Grid oscillators," *Int. J. Infrared and Millimeter Waves,* vol. 9, no. 7, pp. 647–654, 1988.

[4] R. M. Weikle II, M. Kim, J. B. Hacker, M. P. DeLisio, Z. B. Popović, and D. B. Rutledge, "Transistor oscillator and amplifier grids," *Proc. IEEE,* vol. 80, no. 11, pp. 1800–1809, 1992.

[5] R. A. York, "Quasioptical power combining techniques," in *Millimeter and Microwave Engineering for Communication and Radar*, J. C. Wiltse, Ed. Bellingham, WA: SPIE, vol. CR54, pp. 63–97.

[6] J. Birkeland and T. Itoh, "A 16-element quasioptical FET oscillator power combining array with external injection locking," *IEEE Trans. Microwave Theory Tech.,* vol. 40, no. 3, pp. 475–481, 1992.

[7] H. S. Tsai, M. J. W. Rodwell, and R. A. York, "Planar amplifier array with improved bandwidth using folded slots," *IEEE Microwave Guided Wave Lett.*, vol. 4, no. 4, pp. 112–114, 1994.

[8] J. S. H Schoenberg and Z. B. Popović, "Planar lens amplifier," in *1994 IEEE MTT-S Int. Microwave Symp. Dig.,* San Diego, CA, May 1994, vol. 3, pp. 429–432.

[9] M. A. Gouker, R. G. Beaudette, and J. T. Delisle, "A hybrid-circuit tile-approach architecture for high-power spatial power-combined transmitters," in *1994 IEEE MTT-S Int. Microwave Symp. Dig.,* San Diego, CA, May 1994, vol. 3, pp. 1545–1548.

[10] J. D. Kraus, *Antennas,* 2nd ed. New York: McGraw-Hill, 1988, pp. 175–188.
[11] *IEEE Standard Test Procedures for Antennas, ANSI/IEEE Std. 149-179.* New York: IEEE and Wiley-Interscience, 1979, pp. 110–112.
[12] C. Chi and G. M. Rebeiz, "A Quasioptical amplifier," *IEEE Microwave Guided Wave Lett.,* vol. 3, no. 6, pp. 164–166, 1993.
[13] M. Kim, J. J. Rosenberg, R. P. Smith, R. M. Weikle, II, J. B. Hacker, M. P. DeLisio, and D. B. Rutledge, "A grid amplifier," *IEEE Microwave Guided Wave Lett.,* vol. 1, no. 11, pp. 322–324, 1991.
[14] T. P. Budka, M. W. Trippe, S. Weinreb, and G. M. Rebeiz, "A 75–115 GHz quasioptical amplifier," *IEEE Trans. Microwave Theory Tech.,* vol. MTT-42, no. 5, pp. 899–901, 1994.
[15] N. Sheth, T. Ivanov, A. Balasubramaniyan, and A. Mortazawi, "A nine HEMT spatial amplifier," in *1994 IEEE MTT-S Int. Microwave Symp. Dig.,* San Diego, CA, May 1994, vol. 2, pp. 1239–1242.
[16] D. R. Gagnon, "Lens-focused measurements for quasioptical components," in *Workshop Notes for Circuit Level Designing and Modeling of Quasioptical Circuits and Systems at the 1994 Int. Microwave Symp.,* San Diego, CA, June 1994; and D. R. Gagnon, "Highly sensitive measurements with a lens-focused reflectometer," *IEEE Trans. Microwave Theory Tech.,* vol. 39, pp. 2237–2240, Dec. 1991.
[17] M. Kim *et al.,* "A 100-element HBT grid amplifier," *IEEE Trans. Microwave Theory Tech.,* vol. 41, no. 10, pp. 1762–1771, 1993.
[18] D. Rutledge *et al.,* "Oscillator and amplifier grids," in *IEEE Int. Microwave Symp. Dig.,* Albuquerque, NM, June 1992, vol. 1, pp. 815–818.
[19] H. J. Kuno, "Analysis of nonlinear characteristics and transient response of IMPATT amplifiers," *IEEE Trans. Microwave Theory Tech.,* vol. MTT-21, no. 11, pp. 694–702, 1973.
[20] L. J. Kushner, "Output performance of idealized microwave power amplifiers," *Microwave J.,* vol. 32, no. 10, pp. 103–116, 1989.

General Formulas for the Method of Lines in Cylindrical Coordinates

Reinhold Pregla

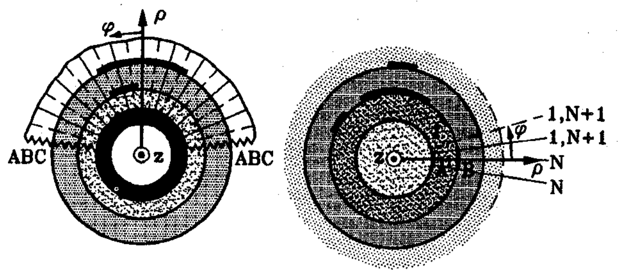
Abstract— **General formulas are given for the method of lines in cylindrical coordinates and angular discretization. They describe the transfer of the fields from one boundary of a cylindrical layer to another in a multilayered structure. With these formulas, programming can be accomplished without performing additional analysis.**

I. Introduction

The method of lines, as a special FDM, enables analytic calculation in a specific direction. In this direction, the structures to be analysed can consist of multiple, stacked layers without causing an increase in the difficulty or complexity of the analysis. In general, field components from the boundary surface of one layer can be transformed to that of another layer. The basic theory and important formulas for this procedure are explained in [1]. These transformation formulas are easily suited to the analysis of waveguides such as those used in integrated optics, [2], [3] and diffused waveguides with up to 80 layers or more can be modeled using this method.

Of late, cylindrical structures have also become more meaningful. The basic principle of using the method of lines to solve wave equations in cylindrical coordinates is given in [4]. A treatment of microstrip lines of arbitrary cross section, accomplished with the help of cylindrical functions, appears in [5], and in [6] the analysis of antennas composed of microstrip and microslot resonators using cylindrical bodies is explained. Dipoles are analyzed in [7] using the methodology described in [1]. A generalized description of the transformation of fields from one cylindrical boundary surface to another, however, has not been completed. The purpose of this document is to provide such a description. Having such general formulas computer programming is made very easy.

Manuscript received October 3, 1994; revised December 31, 1994.
The author is with Fern Universitaet in D-58084 Hagen, Germany.
IEEE Log Number 9412057.



(a) (b)

Fig. 1. Cross sections of cylindrical multilayer structures with discretization lines using ABC's (a) and PBC'c (b). (a) General cross section, (b) sectorial cross section.

II. Methods of Analysis

The general method of analysis, described below, applies to structures such as those diagrammed in Fig. 1. The number of layers in these structures is arbitrary. An arbitrary number of metallic strips or cylinders can be placed between the layers of the structure, and the layers can begin at $\rho = 0$ and extend to infinity. Structures with a ρ-dependent permittivity (graded index fibers) can be successfully modeled by a sufficient number of distinct layers. The goal of this document is thus the formulation of a general transfer for fields between two boundary layers, i.e. from a surface A to a surface B in the ith layer. The procedure for this is analogous to those in [1] and [8], [9], but in cylindrical coordinates and with angular discretization. The permittivities in the layers can also be complex.

The whole field may be obtained from the components in the z direction, e_z and h_z. These are the only Cartesian components in the cylindrical coordinate system, and for these components the following wave equations are valid

$$\overline{\nabla}_t^2 F_z + \frac{\partial^2 F_z}{\partial \overline{z}^2} + \varepsilon_r F_z = 0 \tag{1}$$

where $F_z = E_z$ or $F_z = \tilde{H}_z = \eta_o H_z, \overline{z} = k_o z, \overline{\rho} = k_o \rho$ and

$$\overline{\nabla}_t^2 = \frac{1}{\overline{\rho}} \frac{\partial}{\partial \overline{\rho}} \left(\overline{\rho} \frac{\partial}{\partial \overline{\rho}} \right) + \frac{1}{\overline{\rho}^2} \frac{\partial^2}{\partial \varphi^2} \tag{2}$$

is the Laplace operator in cylindrical coordinates.k_o and η_o are the wave number and wave impedance of free space, respectively. In the following we assume propagation in the z direction. Therefore we write $-j\sqrt{\varepsilon_{re}}$ for $\partial/\partial\overline{z}$. ε_{re} is the effective dielectric constant.

For the solution of the wave (1) and for the determination of the field components, a discretization in φ direction is performed [4]. As stated there, in principle, the analysis is the same as in Cartesian coordinates. Therefore, all that is known for the discretization in cartesian coordinates can be used here for the φ direction. In Fig. 1 two different possibilities are shown. To save memory and computational effort, absorbing boundary conditions (ABC) are suitable (a). If the whole cross section is of interest, periodic boundary conditions (PBC)